2015 John O'Bryan Mathematical Competition
## Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. **Exact** answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. When twice a number is increased by 7, the result is 2015. Find the number.

2. If x represents a negative number, how many of the following quantities **must** be positive? Note that each of the five boxes below contains a different quantity.

$0.5x$	$x - 3$	$3 - x$	$(-2)x$	$|x| + 0.5$

3. Two students in Austin's class failed algebra. Three students in Holden's class failed geometry. There were 11 students in Austin's class and the failure rate in Holden's class was 20%. Assuming that no student was enrolled in both classes, how many students passed one of the classes?

4. In the diagram, $\overline{EB} \perp \overleftrightarrow{AC}$ and points A, B, and C are collinear. $\overrightarrow{BF}$ bisects $\angle GBE$. If $\angle EBF = 28°$ and $\angle CBD = 44°$, find the degree measure of $\angle DBG$.
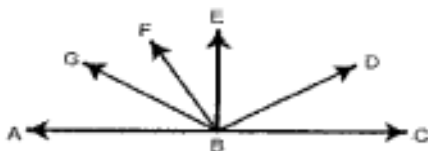
5. A bag contains four squares and one equilateral triangle with side-lengths as indicated below. If two distinct polygons of the five are selected at random, find the probability that both polygons selected have a perimeter greater than 25. Express your answer as a common fraction reduced to lowest terms.

6. Let $A = \{5,10,15,20,25,30,35\}$. Let x be the sum of the seven remainders when each element of A is divided by 4. Let y be the sum of the seven remainders when each element of A is divided by 3. Find the value of $(x+y)$.

7. The price of a gold chain varies directly as the length of the chain. If a gold chain that is 14 inches long costs $266, find the number of dollars in the cost of a gold chain that is 18 inches long.

8. If the smaller base of a trapezoid is increased by 14 and the larger base is decreased by 8, by how much is the median increased or decreased? Write an increase as a positive number and a decrease as a negative number.

9. A right triangle having legs of lengths $10\sqrt{3}$ and $24\sqrt{3}$ is inscribed in a circle of area $A\pi$. Find the value of A.

10. A group of four people are meeting for lunch today in a restaurant. The first person eats lunch there every 8 days; the second person every 9 days; the third person every 10 days; and the fourth person every 12 days. How many days will pass until all four will again be eating lunch in this restaurant on the same day?

11. Leah had x dollars on the table that she wished to give to various charities. She found that if he gave $65 to each charity, she would be $12 short. So she decided to give $62 to each of the same charities and found that she had $48 left over. Find the value of x.

12. Given the set: $\{5, x, 2y\}$ where x and y represent positive integers. The average of the three members of this set is 6. Find the largest possible value of the product of x and y.

13. The ratio of the area of an equilateral triangle to the area of a square is $16\sqrt{3} : 49$. What is the ratio of the perimeter of the equilateral triangle to the perimeter of the square? Give your answer in the form $a:b$ where a and b are integers with no common factors.

14. On a flat surface, Katie starts out by going 7 miles due north, then 6 miles due northwest, then 3 miles due west, and finally 2 miles due southwest. At the end of her journey, how many miles is Katie from her starting point? Give your answer as a decimal rounded to the nearest hundredth.

15. Given A, B, D and A, C, E are sets of collinear points. $\overline{BC} \parallel \overline{DE}$, $AB = x - 1$, $BC = 18$, $AC = 2x + 3$, $BD = y$, $CE = 3y + 1$, and $DE = 24$. Find the ordered pair (x, y). If necessary, express values as reduced fractions.
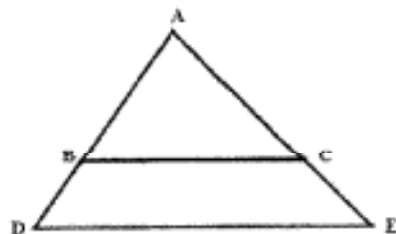
16. One of the interior angles of a convex decagon measure $23°$. What is the maximum number of interior angles of that convex decagon that could be supplementary to the $23°$ angle?

17. The marked price of a computer is discounted by 15% from its retail price. Later, the marked price is discounted 10% to a sales price. Finally, the sales price is discounted 30% to a clearance price of $535.50. Find the original retail price.

18. Working at a constant rate, Jayden would need 30 hours to build a certain brick wall. Working at a constant rate, Cody would need 21 hours to build the same wall. Due to magical chemistry, if they work together, a total of 8 more bricks will be laid per hour than if working alone. By working together, they took exactly 11 hours and 40 minutes to build the wall. Find the number of bricks in the wall.

19. If $\dfrac{5+\sqrt{3}}{3-\sqrt{3}} = \dfrac{a+b\sqrt{c}}{d}$ where a, b, c, and d are integers and $d > 0$, find the minimum value of the product $abcd$.

20. Circle A has center $(6, -2)$ and radius 5. Circle B has center $(-3, 4)$ and radius 3. Find the exact length of the common internal tangent between circles A and B. Express your answer as a radical in reduced form.

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1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

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10. _____

11. _____

12. _____

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14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

20. _____

Name: _____ANSWERS_____   Team Code: _____

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Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. _____**1004**_____

2. _____**3**_____

3. _____**21**_____

4. _____**102**_____   Degrees optional

5. _____**3/5**_____   Must be this reduced fraction.

6. _____**20**_____

7. _____**342**_____   Dollar sign optional

8. _____**3**_____

9. _____**507**_____

10. _____**360**_____

11. _____**1288**_____   Dollars optional

12. _____**21**_____

13. _____**6:7**_____   Must be in this form

14. _____**13.10**_____   Must be exactly this form

15. _____$\left(3, \dfrac{2}{3}\right)$_____   Must be exactly this form

16. _____**8**_____

17. _____**1000**_____

18. _____**1680**_____

19. _____**324**_____

20. _____$\sqrt{53}$_____   Must be in this form

Awards Lists and Solutions to the Team Competition may be found at
http://math.nku.edu/job