2014 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

Calculators may not be used on the first four questions

1. Let $ABCDEF$ be a regular hexagon with sides of length 2. Suppose that k is the degree measure of $\angle AFB$ and m is the distance between segments $\overline{AB}$ and $\overline{DE}$. Find the exact value of km^2.

2. Let k be the coefficient on the term x^5y^3 in the expansion of $(x+y)^8$. Let πw be the area of a circle inscribed in a square that has area 36. Find $k + w$.

3. Suppose $\log(k)-\log(3)+\log(5)=2$ and w is the absolute value of the difference between the roots of the equation $x^2-4x-21=0$. Find the value of kw.

4. Let $m=\begin{vmatrix}3 & 1\\ 2 & 4\end{vmatrix}-\begin{vmatrix}8 & 4\\ 3 & 1\end{vmatrix}$ and let $4^{(n+7)}=\left(\frac{1}{16}\right)^6$. Find the value of $m + n$.

Calculators may be used on the remaining questions

5. Let (x,y) be the solution to the system $\begin{cases}5x+2y=1071\\ 13x-y=3851\end{cases}$. Let k be the length of the shortest side of a triangle whose perimeter is 20424 and whose sides have lengths in the ratio 89:786:827. Find the value of $(x+y+k)$.

6. Let x be the smallest positive integer such that $x!(x+2)!=(x+3)!$ Let y be the length of the longest altitude of a triangle with sides of length 11, 13, and 20. Find the value of the product xy.

7. Bob has two sequences – one arithmetic and the other geometric. The sum of the first 16 terms of his arithmetic sequence is 1608; its first two terms are $2k+4$ and $2k+7$. His geometric sequence has w as its first term, sums to 25 and has a common ratio of 0.2. Find $k + w$.

8. Let $B = \{2,3,5,12,29\}$. Define p to be the probability that the sum of two distinct elements of B selected at random is a prime integer. Define q to be the probability that the sum of three distinct elements of B selected at random is <u>not</u> a prime integer. Find the value of $p + q$. Express your answer as a common fraction.

9. (T1) Find the product of the smallest and largest integer values of x that satisfy $-33<-2(x-1)-3<3$

10. (T2) Four red flags and six blue flags are to be arranged on a pole. If all flags are identical except for color, in how many *distinct* ways can the flags be arranged from top to bottom?

Names: _____ Team Code: _____

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Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

	ANSWER	SCORE
1.	_____	_____
2.	_____	_____
3.	_____	_____
4.	_____	_____
5.	_____	_____
6.	_____	_____
7.	_____	_____
8.	_____	_____
T1.	_____	_____
T2.	_____	_____

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.

Note: Questions (9) and (10) will be used to break ties for positions 1, 2, and 3. If a tie remains after two tiebreaker questions, the tie will be broken if possible using (a) Total # correct answers, (b) Total # firsts, (c) student total scores on the individual exams.

TOTAL SCORE

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Answers for the Two-Person Speed Event

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1. 360

2. 65

3. 600

4. -5

5. 1179

6. 36

7. 57

8. 9/10 Must be this fraction

T1. -15

T2. 210

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