**2013 John O'Bryan Mathematical Competition**
**Junior-Senior Individual Test**

**Directions:** Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. If $x^4 - x^2 - 20 = 0$, how many of the distinct roots are non-real?

2. Let $i = \sqrt{-1}$. If $(3i)(ki) = -12$, find the value of $k$.

3. Let $k$ be an integer such that $30 < k < 45$. Find the sum of all distinct values of $k$ such that $k$ is **not** the sum of 2 or more consecutive **odd** natural numbers.

4. Find the sum of all distinct positive integer multiples of 7 if each multiple of 7 is less than 2013.

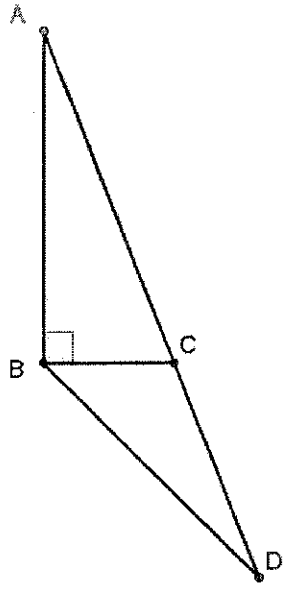
5. If $x < 0$, then find the value of $x$ such that $|2 - 3x| = 23$.

6. One circle has an equation of $(x-8)^2 + (y+2)^2 = 25$. A second circle has an equation of $2x^2 + 2y^2 + 12y = 1$. Find the slope of a line that is perpendicular to the line that joins the centers of the two given circles.

7. The circumference of a circle is $38.74\pi x$. The area of this same circle is $kx^2$. Find the value of $k$ rounded to the nearest integer.

8. An observer in a building that rises vertically notes that two objects on a horizontal road below have respective angles of depression of 26° and 19° respectively. If the distance from the base of the building to the far object is 1827 feet and if the horizontal road runs directly away from the observer, find the distance (in feet) between the two objects rounded to the nearest integer.

9. In the diagram (not necessarily drawn to scale), points $A$, $C$, and $D$ are collinear. $AB = 12$, $BC = 5$, $\angle CBD = 45°$, and $\overline{AB} \perp \overline{CB}$. Find the length of $\overline{CD}$. Express your answer as an improper fraction reduced to lowest terms.



10. It is known that 475 logs are piled in such a way that each row has one more log than the row above. If the top row has as few logs as necessary to stack all 475 logs, find the number of logs in the bottom row.

11. Let $e$ be the base for natural logarithms and let ln be the symbol for natural logarithms. Find, written as a single fraction in terms of $e$, the value of $x$ such that $\ln(x)-\ln(x-1)=1$.

12. Find the smallest positive integer that is an integral multiple of 18 distinct positive integers and is 4 less than the square of a positive integer.

13. Let $C(n,k)=\dfrac{n!}{k!(n-k)!}$ where $n$ and $k$ represent positive integers. Find the **sum** of all distinct values of $n$ such that $7<C(n,4)<106$.

14. If $A=2\begin{bmatrix}2 & 3\\ 1 & -3\end{bmatrix}-3\begin{bmatrix}5 & 4\\ 1 & 7\end{bmatrix}$, find the value of $\det(A)$.

15. A drawer contains 3 black socks, two brown socks, and two blue socks. Two socks are drawn at random from this drawer without replacement. Find the probability that the two socks drawn were of the same color. Express your answer as a common fraction reduced to lowest terms.

16. Christy bought a truckload of cantaloupe for \$405.81, and Marnie bought a truckload of cantaloupe for \$379.42 to stock their vegetable stand. There were 889 cantaloupes in Christy's truckload, and there were 881 cantaloupes in Marnie's truckload. Find the average number of **cents** paid per cantaloupe. Round your answer to the nearest hundredth of a cent.

17. Find the sum of all distinct negative integer solutions for $x$ satisfying $\log_{\left(\frac{1}{2}\right)}(x+15)\le 0$.

18. Find the circumference of the circle whose equation is $x^2+y^2-\dfrac{3}{2}x-\dfrac{5}{2}y+\dfrac{3}{2}=0$. Round your answer to the second decimal place.

19. A box is formed by removing squares of side length $x$ from the corners of a rectangular piece of sheet metal 8.5 feet wide and 10.25 feet long. If the resulting box is to have a volume of at least 40 cubic feet, then $k\le x\le w$. Find the value of $(k+w)$ rounded to the second decimal place.

20. In the addition alphametric shown to the right, each letter stands for the same digit throughout the puzzle. No digit is represented by more than one letter. For your answer write the 4 digits (from left to right in order) that represent ROOM.

$$\begin{array}{rcccc} & & & A & T \\ + & & M & A & R \\ \hline & R & O & O & M \end{array}$$

Name: _____ANSWERS_____ Team Code: _____

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## Junior-Senior Individual Test

**Note:** All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. 2
2. 4
3. 266
4. 289296
5. –7
6. –8
7. 1179
8. 537
9. $\frac{65}{7}$ Must be this reduced improper fraction
10. 31
11. $\frac{e}{e-1}$ or $\frac{-e}{1-e}$ Must be a single fraction
12. 252
13. 21
14. 291
15. $\frac{5}{21}$ Must be this reduced common fraction
16. 44.36
17. –105
18. 4.97
19. 3.33
20. 1009