2012 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Set $A = \{1,2,3,4,5\}$. If one member of Set A is selected at random and substituted for x in the expression $\frac{2x+1}{3}$, find the probability that this expression will then have a value that is an integer. Express your answer as a common fraction reduced to lowest terms.

2. Consider the statement: *If the largest of four different positive integers is less than 8, then the average of the four positive integers is more than 6.* Is this statement **ALWAYS** true, **SOMETIMES** true, or **NEVER** true? For your answer write the whole word: **ALWAYS, SOMETIMES,** or **NEVER** – whichever is correct.

3. A line segment has endpoints at (14,9) and (2,37). Find the **ordered pair** that represents the midpoint of that segment.

4. Does the point (4,7) lie **INSIDE, OUTSIDE,** or **ON** the circle whose equation is $x^2+(y-2)^2=42$? For your answer, write the whole word: **INSIDE, OUTSIDE,** or **ON** – whichever is correct.

5. A trapezoid has sides with respective lengths 2, 41, 20, and 41. Find the length of an altitude of this trapezoid.

6. Find the maximum possible value of $4y$ if x and y must be positive integers satisfying the system:

$$\begin{cases} x+y \le 16 \\ y-x \le 6 \end{cases}$$

7. Today there are k dollars in a bowl. Every 5 days from today, \$6 will be added to the bowl. Every 8 days from today, \$9 will be removed from the bowl. After the addition and/or subtraction is made from the bowl on the day which is 365 days from today, there will be \$74 in the bowl. Find the value of k.

8. The circumference of a circle is $C\pi$ meters. In this circle a chord of length 26 meters is 5 meters from the center of the circle. Find C.

9. Consider the statement: *The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices of the right triangle.* Is this statement **ALWAYS** true, **SOMETIMES** true, or **NEVER** true? For your answer write the whole word: **ALWAYS, SOMETIMES,** or **NEVER** – whichever is correct.

10. In number base k, 2666 has the same value as the base five number 31004. Find k.

11. Let p be a positive integer multiple of 8. Let $A = \{p, 2p, 3p, 4p, p+2. p+3. p+4. p+24. \frac{1}{2}p, \frac{1}{3}p\}$. What is the probability that a random draw from the set A will be divisible by 8 without remainder?

12. In rhombus $ABCD$, the degree measure of angle DAB is 60°. A circle passes through vertices A, B, and D and intersects diagonal AC at E. If $AE = 9$, find the perimeter of the rhombus. Give your answer as a decimal rounded to two places.

13. Suppose that at a certain high school, 5% of female students and 4% of male students are on the math team. If 25/41 of the students at this high school are male and there are 27 students on the math team, find the total number of students at the high school.

14. Penny has three times as many quarters as she has dimes. If the total value of her quarters and dimes is $4.25, find the total number of coins that Penny has.

15. Find the exact area of an equilateral triangle if the perimeter is 6. Leave your answer as a radical expression in simplest form.

16. Mowing at constant rates, Alfred can mow the whole lawn in 6 hours; Betty can mow the entire lawn in 4 hours, and Chad can mow the entire lawn in 3 hours. Assuming no loss of efficiency, find the number of **minutes** it will take before the lawn is mowed if all three start mowing the lawn at the same time.

17. A classroom in the shape of a rectangular solid has a volume of 7840 cubic feet. If the height of the classroom is half the length and the width is 8 feet less than the length, what is the height of the classroom?

18. How many of the following five statements are true for all real numbers x and y?

    a) $|x| = -|x|$
    b) $|y-3| = |3-y|$
    c) $|y-5x| = |5y-x|$
    d) $|x+y| \le |x| + |y|$
    e) $|x-y| \le |x| + |y|$

19. Find the smallest 3-digit positive integer that meets <u>**all**</u> three of the following conditions:

    a) If two is added to the number formed by any two of its digits (in either order) the result is prime.
    b) If two is added to the number formed by its three digits (in any order) the result is prime.
    c) At least two of the digits in the number must be different.

20. The points $(7,k)$ and $(11,w)$ lie on a circle whose equation is $x^2 + y^2 = p$. If k and w are both positive integers with $k + w < 132$, find the sum of all distinct values of p.

Name: ______ANSWERS______ Team Code: _______

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1.	2 / 5		11.	1 / 2 or 0.5	
2.	NEVER	Must be Full Word	12.	31.18	
3.	(8,23)	Must be Ordered Pair	13.	615	Students Optional
4.	INSIDE	Must be Full Word	14.	20	Coins Optional
5.	40		15.	$\sqrt{3}$	Must be Exact
6.	11		16.	80	Minutes Optional
7.	41	Dollars Optional	17.	14	Feet Optional
8.	26	Meters Optional	18.	3	
9.	ALWAYS	Must be Full Word	19.	177	
10.	9	Or "Base 9"	20.	710	

Awards Lists and Solutions to the Team Competition may be found at http://www.nku.edu/~math/job/